Seat No. : _______________

OA-138

October-2018

M.Sc., Sem.-II

407 : MATHEMATICS
Differential Geometry-I
(Old) (Repeater)

Time : 2:30 Hours] **[Max. Marks : 70**

1. (A) Find parametrizations of the following level curves :
   (i) $x^2 + y^2 = 1$ **7**
   (ii) $x^2 - y^2 = 1$ **7**

   OR

   Find the Cartesian equations of the following parametrized curves :
   (i) $r(t) = (\cos(t), -\sin(t))$
   (ii) $r(t) = (e^t, t^3 + 1)$

   (B) Answer any **four** : **4**
   (1) Let $r(t) = \left(\frac{3}{5}\cos(t), -\sin(t), \frac{-4}{5}\cos(t)\right)$. Is r a unit speed curve ?
   (2) Sketch the curve $r(t) = (\cos(t), 2\sin(t))$.
   (3) Sketch the curve $r(t) = (e^t\cos(t), e^t\sin(t))$
   (4) Sketch the curve $r(t) = (t, \sin(t))$.
   (5) Let $r(t) = (t, \cos h(t))$. Calculate the arc length starting at point (0, 1).
   (6) Let $r(t) = (t, t^2, t^3)$. Find the angle between $r(1)$ and the tangent vector $\dot{r}(1)$.

2. (A) (1) Compute the curvature k and the torsion τ for the curve $r(t) = \left(\frac{4}{5}\cos(t), \frac{4}{5}\sin(t), \frac{3}{5}t\right)$. **7**
   (2) Find the torsion τ for the curve. **7**
   $r(t) = (t, t^3, t^4)$. Show that it is not a planar curve.

   OR

   (1) Compute the curvature k and the torsion τ for the curve $r(t) = \left(\frac{1}{3}(1+t)^{\frac{3}{2}}, \frac{1}{3}(1-t)^{\frac{3}{2}}, \frac{t}{\sqrt{2}}\right)$.
   (2) Find the torsion τ for the curve $r(t) = (t, t^2, t^3)$. Show that it is not a planar curve.

(B) Answer any **four** : **4**

(1) Define the signed curvature of a unit speed curve r(s) in $\mathbb{R}^2$.

(2) Let $r(t) = (2, t, t^2)$. Is the curve r a planar curve ?

(3) Give an example of a curve whose curvature is zero at every point. (Do not prove).

(4) Give an example of a curve whose curvature is 1 at every point. (Do not prove)

(5) Write down (without proof) a formula for the curvature of r(t).

(6) Write down (without proof) a formula for the torsion of r(t).

3. (A) (1) Show that the level surface $\frac{x^2}{2^2} - \frac{y^2}{3^2} - \frac{z^4}{4^2} = 1$ is a smooth surface. **7**

(2) Find the equation of the tangent plane to the surface $x^2 + y^2 + z^2 = 1$ at the point $(0, 0, -1)$. **7**

OR

(1) Consider the surface patch $\sigma(u, v) = (u, v, u^2 + v^2)$. Find the equation of the tangent plane at the point $(1, 1, 2)$.

(2) Show that the set $S = \{(x, y, 0) \mid x^2 + y^2 < 1\}$ is a regular surface.

(B) Answer any **three** : **3**

(1) Give (without proof) a parametrization $\sigma(u, v)$ of the unit sphere $x^2 + y^2 + z^2 = 1$.

(2) Show that the line $r(t) = (1, t, -t)$ is contained in the surface $x^2 + y^2 - z^2 = 1$.

(3) Give (without proof) an example of an unbounded surface.

(4) Give (without proof) a tangent vector to the sphere $x^2 + y^2 + z^2 = 1$ at the point $(0, 0, 1)$.

(5) Parametrize the surface $2x + 3y - z = 1$ (Do not prove)

4. (A) (1) Define a surface of revolution. Show that the surface $x^2 + y^2 - z^2 = 1$ is a surface of revolution and find a parametrization for this surface. **7**

(2) What kind of quadric is S, where S is given by the equation $\frac{x^2}{2^2} + \frac{y^2}{3^2} + \frac{(z-1)^2}{4^2} = 1$. **7**

OR

(1) Determine the area of the part of the paraboloid $z = x^2 + y^2$ with $z \leq 4$.

(2) Calculate the first fundamental form of the surface $\sigma(u, v) = (u, v, u^2 - v^2)$.

(B) Answer any **three** : **3**

(1) Define a ruled surface.

(2) Give an example (without proof) of a triply orthogonal system.

(3) Define an isometry.

(4) Define a conformal mapping.

(5) Give (without proof) an isometry from the unit sphere to itself, other than the identity.

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OA-138

October-2018

M.Sc., Sem.-II

407 : (Mathematics)
(Metric Spaces)
(New Course)

Time : 2:30 Hours] **[Max. Marks : 70**

1. (A) Answer the following questions : **14**

(1) Let (X, d) be a metric space, prove that there is a bounded metric on X which is equivalent to d.

(2) Let $U = \{(x, y) \in \mathbb{R}^2 \; x \notin Z, y \notin Z\}$. Is U open in $\mathbb{R}^2$? Find the interior points of U.

OR

(1) Prove that a non-empty open set in $\mathbb{R}$ is the union of countable collection of pairwise disjoint open intervals.

(2) Let $(U) = \{(x, y) \in \mathbb{R}^2 / xy = 0\}$. Is U open in $\mathbb{R}^2$? Find the interior points of U.

(B) Attempt any **four** : **4**

(1) Show that every open ball is an open set.

(2) The set Q of rationals is open in R. True of False ? Justify.

(3) Determine all the open sets in the discrete metric space (X, d).

(4) Define the interior and the closure of a set A in a metric space X.

(5) Draw the open ball B(0, 1) in $\mathbb{R}^2$ with norm p = 1.

(6) Draw the open ball B(0, 1) in $\mathbb{R}^2$ with norm $p = \infty$.

2. (A) Answer the following questions : **14**

(1) Prove that the limit of a sequence in a metric space is unique.

(2) Define Cauchy sequence. Prove that every Cauchy sequence is bounded.

OR

(1) Prove that a subset E of a metric space (X, d) is closed iff E contains all its limit points.

(2) When we say that D is dense in $\mathbb{R}$? Give two open-dense subsets of $\mathbb{R}$.

(B) Attempt any **four** : **4**

(1) Prove that every constant sequence is convergent.

(2) Give an example of a bounded sequence that is not a Cauchy sequence.

(3) What are the limit points of Q ?

(4) Is it true that the set Q^c of irrational numbers is dense in $\mathbb{R}$? Justify.

(5) When we say that A is a bounded subset of a metric space ? Explain.

(6) Give two unbounded dense open subsets of $\mathbb{R}^2$?

3. (A) Answer the following questions : **14**

(1) Let f be a function from metric space X to a metric space Y and $x \in X$. If f is continuous at x then prove that given an open set V containing f(x), there exists an open set U such that $f(U) \subset V$.

(2) Let $A = \{(x, y) \in \mathbb{R}^2 / \cos(x^2) + x^3 - 47y > e^x - y^2\}$. Give details to show that A is open in $\mathbb{R}^2$.

OR

(1) Let X and Y be metric spaces and f and g are continuous functions from X and Y then show that the set $(x \in X / f(x) \neq g(x)\}$ is open in X.

(2) Define uniform continuous function. Give an example of a continuous function that is not uniform. Give details.

(B) Attempt any **three** : **3**

(1) If f is a function defined on the discrete space, show that f is continuous.

(2) Show that every constant function is continuous.

(3) Define a homeomorphism. Give an illustration.

(4) Give an example of a continuous function from $\mathbb{R}^2$ to $\mathbb{R}$.

(5) True or False : The function $f(x) = \sin x$ is uniformly continuous on $\mathbb{R}$.

4. (A) Answer the following questions : **14**

(1) Let X be a connected metric space f be a continuous map from X to Y, prove that f(X) is connected.

(2) Prove that R is not compact with the standard metric.

OR

(1) Prove that any compact subset of a metric space is closed and bounded.

(2) Prove that R is connected with the standard metric.

(B) Attempt any **three** : **3**

(1) Prove or disprove : The unit sphere $U = \{x \in \mathbb{R}^2 / \| x \| = 1\}$ is compact.

(2) Prove that set $X = \{x_1, x_2, \ldots x_n\}$ is a compact subset of $\mathbb{R}$.

(3) Show that Z, the set of integers is not a compact subset of $\mathbb{R}$.

(4) Prove or disprove : The discrete metric space is compact.

(5) Show that $A = \left\{\frac{1}{n} / n \in N\right\} \cup \{0\}$ is a compact subset of $\mathbb{R}$.